M-16-A

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

May/June 2016 End Semester Examinations
B.Tech. (Marine Engineering) – 2009 batch onwards

Semester I - Mathematics – I (UG21 T1102 / UG21T2102)

Date :11.06.2016 ~~2nd Setup best Supply~~ **Max. Marks: 100**
Time: 3 Hrs **Pass Marks :50**

Part – A **(3 X 10 = 30 Marks)**
Compulsory Question

1) a) Find the asymptotes of the curve
$y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0.$

b) Find the extremals of the functional $\int_{x_0}^{x_1} \left(\frac{y'^2}{x^3}\right) dx$

c) If A $= \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, show that AA^* is a Hermitian matrix, where A^* is the conjugate transpose of A.

d) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4}\, dz$, where C is the circle $|z| = 2$.

e) If $y = \dfrac{x^2}{(x-3(x-1)(x-2)}$ then find y_n

f) Find the saddle points of the function. $x^3+y^3-3x-12y+10$

g) Show that $\int_0^1 \frac{x^2dx}{(1-x^4)^{\frac{1}{2}}} \times \int_0^1 \frac{dx}{(1+x^4)^{\frac{1}{2}}} = \frac{\pi}{4\sqrt{2}}$

by applying $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx = \pi$

h) Compute the value of $\iint_R ydxdy$ where R is the region in the first quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

i) Determine the constant 'a' so that the vector

$\vec{v} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal $\nabla \cdot V = 0$

j) Evaluate $\left[\dot{\vec{r}} \;\; \ddot{\vec{r}} \;\; \dddot{\vec{r}}\right]$ *when* $\vec{r} = a\cos u\hat{i} + a\sin u\hat{j} + bu\hat{k}$

(k) Find the eigen values of the matrix A $= \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$

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Part – B

(5x 14=70 Marks)

Answer Any Five of the following

2) a) If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that
$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0.$ (6 Marks)

b) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2+y^2+z^2 = 1$ (8 Marks)

3) a) Solve$(y\, logy)dx + (x - logy)dy = 0.$ (6 Marks)

b) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$. prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ and
$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4\cos^3 u}.$ (8 Marks)

4) a) Find the curves on which the functional $\int_0^1 \left[y'^2 + 12xy\right] dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremised. (8 Marks)

b) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$ (6 Marks)

5) a) Evaluate $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right\}^{1/n}$ (7 Marks)

b) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dxdydz$ (7 Marks)

6) a) Find a unit Vector normal to the surface $xy^3z^2 = 4$ at the point (-1,-1, 2). (6 Marks)

b) If r is the distance of a point (x,y,z) from the origin, prove that $Curl\left(k \times grad\frac{1}{r}\right) + grad\left(k.grad\ \frac{1}{r}\right) = 0$, where K is the unit vector in the direction OZ. (8 Marks)

7) a) Expand $f(z) = \frac{1}{\{(z-1)(z-2)\}}$ in the region :

i) $|z| < 1,$

ii) $1 < |z| < 2,$

iii) $|z| > 2,$

iv) $0 < |z-1| < 1.$ (8 Marks)

b) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}dz$,where C is the circle $|z| = 3$. (6 Marks)

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$\cancel{x^2+y^2+z^2-2xy-2yz}$

(xyz)

fgh

8) a) Find the characteristic equation of the matrix. (8 Marks)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Verify Caley – Hamilton theorem and hence evaluate the matrix equation.

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 = 5A^3 - 8A^2 + 2A - I$$

b) Discuss the consistency of the following system of equation. (6 Marks)

$$2x + 3y + 4z = 11$$
$$x + 5y + 7z = 15$$
$$3x + 11y + 13z = 25$$

If found consistent, Solve it.

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$y \log y \, dx = (\log y - x)\, dy$

$\dfrac{y \log y}{\log y -}$